Learning with Interactive Virtual Math in the Classroom
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Proceedings of the 13th International Conference on Technology in Mathematics Teaching

ICTMT 13

École Normale Supérieure de Lyon / Université Claude Bernard Lyon 1

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Gilles Aldon, Jana Trgalová
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Chapter 1

Introduction

The 13th International Conference on Technology in Mathematics Teaching – ICTMT 13 was organized by the Ecole Normale Supérieure de Lyon and the University Lyon 1. It was held in Lyon, France, 3 to 6 July, 2017.

This biennial conference is the thirteenth of a series which began in Birmingham, UK, in 1993, under the influential enterprise of Professor Bert Waits from Ohio State University. The last conference was held in Faro, Portugal, in 2015 and the next conference will be held in Essen (Germany) in July 2019.

The ICTMT conference series is unique in that it aims to bring together lecturers, teachers, educators, curriculum designers, mathematics education researchers, learning technologists and educational software designers, who share an interest in improving the quality of teaching and learning, and eventually research, by effective use of technology. It provides a forum for researchers and practitioners in this field to discuss and share best practices, theoretical know-how, innovation and perspectives on educational technologies and their impact on the teaching and learning of mathematics, as well as on research approaches.

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The general theme of this conference is related to the progress of mathematics education research on the design and integration of technology in educational settings, for learners of all ages from primary schools to universities.

The ICTMT 13 gave to all participants the opportunity to share research and to report progress regarding technology in the mathematics classroom. The following themes were presented and discussed during the four days of the conference and these proceedings are the result of both the proposals and the discussion made during the presentation slots.
Curriculum

Technology and its use impact the ways that the mathematics curriculum is designed and implemented both in schools and at the university level. What are the new impacts of technology on the content, progression and approach to the mathematics curriculum?

Assessment

Technology offers through its functionalities and affordances new possibilities for assessment in mathematics and particularly for formative assessment. How can teachers support the students’ learning that make use of these functionalities and affordances? How can technology support students to gain a better awareness of their own learning?

Students

Does technology still motivate students to learn mathematics? How can technology support students’ to learn mathematics? How can technology foster the development of creative mathematical thinking in students? How can students use their day-to-day technological skills/experiences to support their mathematics learning in and out of schools?

Teachers

Technology can provide a means for mathematics teachers’ professional development through online professional development initiatives, such as blended courses and more recently “massive Open Online Courses (MOOCs). How can technology best support mathematics teachers’ professional development? What are the design principles for technology-mediated professional development courses? How can the impact of such courses on mathematics teachers’ professional learning be assessed? Does the use of technology within professional courses for practicing mathematics teachers impact positively on teachers’ uses of technology in mathematics lessons?

Innovation

New developments in technology for learning and teaching mathematics come both from the design of new applications and from research and innovation. In what ways can these developments enhance mathematics teaching and learning? How can technology become a bridge between mathematics and other subjects? Does creativity in the design of technology impact the creativity of students in maths classes?
Software and applications

What is new in the design of educational software and applications? How can the recent technological developments, such as robotics, touch technology, virtual reality, be exploited to refresh or enhance mathematics teaching and learning?

The plenaries of this ICTMT 13 are available on: https://ictmt13.sciencesconf.org/resource/page/id/16
Interactive Virtual Math (IVM) is a visualization tool to support secondary school students’ learning of dynamic functions situations graphs. The logbook-function allows teachers to get continuous and real-time assessment on classroom progress and of individual students’ learning process. In a teaching experiment involving four mathematics teachers and their students, we investigated how the tool was used by the students and by the teachers.

Keywords: visualization, Virtual Reality, interactive tool, secondary education, learning analytics

AIM AND RESEARCH BACKGROUND

Students’ difficulties with tasks involving dynamical situations are well documented in the literature. And there is also a body of knowledge that shows that conventional curricula have not been effective in promoting covariational reasoning in students (Carlson, Larsen, & Lesh, 2003). New technologies can allow for studying dynamic events and therefore be valuable for students to analyse and interpret dynamic function situations. The aims of the Interactive Virtual Math-project are to design and develop a digital tool for learning covariation graphs at high school (14-17 years old students) and to explore the use of new technologies for learning in classroom. The project started in 2016 as a proof of concept in which a prototype tool was developed and tried out with 14-15 years old students (Palha and Koopman, 2016). In the present stage we explore how the tool is used in classroom by teachers.

INTERACTIVE VIRTUAL MATH

Research provides some directions to develop instruction that supports the learning of covariational reasoning. Thompson (2011) states that it is critical that students first engage in mental activity to visualize a situation and construct relevant quantitative relationships prior to determining formulas or graphs. Also, learners should be helped to focus on quantities and generalizations about relationships, connections between situations, and dynamic phenomena. Digital tools can be valuable for students to analyse and interpret dynamic functional situations. These experiences, when connected to proper curriculum materials and teacher support, can become rich opportunities for students to learn covariational reasoning (Carlson et al, 2003). Tools that include Educational Data Mining (or learning analytics) also have the possibility to generate new understandings of how students learn and how to adapt our environments to those new understandings (Berland, Baker, & Blikstein, 2014). Following these ideas, the IVM-tool was designed and developed to (i) help learners to focus on the relevant quantitative relationships and engage them in the mental activity of visualizing these relationships; (ii) help teachers to get more data about students processes while solving covariation problems. The tool and an instructional video about how it works can be respectively found at https://virtualmath.hva.nl (select EN for English) and https://youtu.be/lc7mNUcZ8CQ.

Students’ visualizing relationships
When entering the tool the students are given a task that encourage them to imagine two variables changing simultaneously. The tool requests the students to construct the graphical representation and the verbal explanation for this relation on themselves within the application. That is, it requires students to represent their concept image graphically and verbally (Vinner, 1983). Through hints and feedback the student is challenged to improve his own construction. The tool also includes the use of Virtual Reality (VR), which is still very limited. The use of VR (sound, movement, interaction) is expected to improve the experience of the graphic situation.

Teacher’s use of data about students’ processes

Another feature of the tool is the logbook-function, which is only available for teachers. Students’ attempts to solve the tasks and whether they view the help-features are recorded and summarized in the logbook. This function allows teachers to get continuous and real-time assessment on the classroom progress and on individual student’s learning process, which can be used by the teacher to provide individual feedback and to orchestrate classroom discussions. It also provides more data about students’ processes while solving covariation problems.

METHOD

Two versions of the prototype have been developed so far. The first version of the prototype was tested with four students. The four students improved their original graphical representation through relating representations and using quantitative reasoning. In the present study we investigate the second prototype version of the tool use in classroom. We conducted a small scale experiment involving four classes and their students and teachers that used IVM during one lesson (45-50 minutes). Because we wanted to explore how students use the tool in the regular classroom practice the teachers were encouraged to setup the lesson from themselves. This paper reports part of the whole study (Palha, 2017). It concerns students’ experiences with the tool and the corpus data consists of students’ responses to questionnaires.

The participants were seventy nine students and four teachers from four classrooms in different schools in The Netherlands: nine students from the first year of the bachelor mathematics teacher, twenty-eight students from 11th grade with, pre-university stream with mathematics B; twenty one students from 10th grade with pre-university stream with mathematics B and twenty one students from 10th grade with, vocational stream with mathematics A. The four classes vary in their mathematical knowledge and ability. It is expected that the 10th grade vocational is the class with less pre-knowledge. No student had, as far as we know, worked before with the tool before the experiment.

The four teachers were invited to take part of the study; they knew about the tool but they were not used to work with it. The teachers are two men and two women with ages varying between 28 and 40 years and with teaching experience varying between 5 to 15 years. The teachers were selected by their teaching experience (we wanted to have a different range of experience since this is a factor that influences classroom performance). And, because they had previously showed interest in using the tool with their students. Not all teachers dare to experiment new approaches especially technological tools that are still in development. We should therefore be careful with the generalization of the results of the experiences of these teachers as they are not representative for the Dutch teachers. More details about the study can be found in Palha (2017).
MAIN FINDINGS

About half of students in all classes reported that the tool have helped them to create, to improve or correct a graph. The way students felt supported by the tool varied per class. Students at 10th and 11th grade with mathematics B reported to have already an idea about the shape of the graph and the tool helped them to work it out and consolidate this idea. Students-teachers at the bachelor have a good idea about the graph and the tool helped them to correct some mistake. The half of the students at the 10th grade following vocational stream also felt support of the tool but they did not have previously any idea about how the graph would be or they had vague idea. The tool helped them in the construction of the graph and to improve their vague initial image.

Specifically, all classes reported that seeing the result of the form of the jar at the end and the self-construction graph were the most helping to them (with exception of one class, in which a slightly higher percentage pointed the help 3D animation as more helpful than the self-construction). Also the comparison feature was considered by the four classes helpful. The help-features were not often mentioned.

All students in the four classes (with one exception in one class) reported that they could work independently with the tool. In three of the four classes a great percentage of the students (81%-89%) reported that they haven't needed help at all and a small percentage reported that they felt the need of some help (11%-19%). In the fourth class (10th grade vocational) about the halve (48%) didn't need help and the other halve (48%) needed some help.

The findings suggest that the students can work independently with the tool in the classroom and without much help. The tool can create opportunities for students to produce and try to improve a mathematical representation of a dynamic event. However, we do not provide much information about the process of coming to generate the graph representations and verbal explanations and its transformation. This study invites further research on this matter. Our research also calls for an extension of the tool and improvement of some features, students provided insightful suggestions that can help us in this direction.

REFERENCES


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