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In most teacher training programs for Dutch mathematics teachers, history of mathematics is a required part of the curriculum. The courses provide historical background knowledge of certain mathematical developments to the students. This knowledge could also affect prospective teachers' views on the nature of mathematics and the pedagogical choices they make for their classrooms. These effects have been examined in a small qualitative research project with two different groups of students from a teacher-training program in Amsterdam. The results are discussed in this paper and can be useful in describing and evaluating the relation between knowledge of history of mathematics and classroom activities.

Keywords: history of mathematics, teacher training, empirical study

Introduction.

I have been teaching history of mathematics courses at the University of Applied Sciences in Amsterdam (in Dutch: Hogeschool van Amsterdam, in short: HvA) for the last eight years. Over time, the goals of the courses I designed have shifted from providing background knowledge of history of mathematics to my students, to showing them what this could mean in terms of pedagogical choices and classroom activities. This assumes that there actually is a relation between the two. In my research I want to focus on this relation, in part to describe it and in part to evaluate it.

Organization of the paper.

This paper is organized in three sections. First the Dutch system of teacher training is briefly described; in particular the way history of mathematics is incorporated. In the first section I also focus on the situation at HvA, the context in which the empirical part of the study took place. Next, the aim of the study and the research method are described in the second section. Finally, the results are presented and discussed. Acknowledgements and references will conclude this paper.

History of mathematics in Dutch teacher training.

To become a mathematics teacher in the Netherlands, there are two main options. (Van den Bogaart et al., 2018) There is the university program, which exists of a three-year undergraduate program in mathematics, followed by a two-year master's program in science education. This route leads to a teaching qualification for all secondary levels in mathematics, for pupils aged 12 to 18. This is called a first degree qualification, or a master's degree in teaching. However, in this paper the focus is on the other option, chosen by the large majority of (future) Dutch mathematics teachers.

The alternative route consists of a four-year program at a university of applied sciences. In these four years mathematics fills about 30% of the curriculum. Students also take courses in pedagogy, psychology, professional skills etc. Practical exercises for teachers (e.g. internships in secondary schools) form a significant part of this program. This leads to a teaching qualification for the lower secondary levels in mathematics (pupils aged 12 to 15). This is called a second degree qualification, or a bachelor's degree in teaching. Once these four years are successfully finished, teachers have the opportunity to advance in another three-year program at a university of applied sciences. In this program, approximately 60% of the curriculum is devoted to mathematics. This finally leads to the full first degree qualification for all secondary levels. This is considered equal to a master's degree in teaching.

Institutions for teacher education have autonomy in designing their curriculum, but the programs are grounded on formal 'knowledge bases' (Kennisbasis in Dutch, KB in short). (Van den Bogaart et al., 2018) There is a KB for Mathematics, a KB French, Biology, etc. There are also generic KBs for pedagogy and other common knowledge for teachers. Since the Dutch system recognizes two degrees of teaching, there is a separate KB Mathematics Bachelor for second degree teachers and another KB Mathematics Master for first degree teachers in programs at universities of applied sciences. I will refer to the two KBs in Mathematics as KBM2 and KBM1. In both KBM1 and KBM2, history of mathematics can be found as a specific subdomain.[1]

KBM2 Domain	
General mathematical techniques	e.g. reasoning and modelling
Calculus	Mainly real valued function calculus in one variable
Geometry	Euclidean geometry, analytic geometry
Algebra	Basic algebraic skills and elementary set theory
Stochastics	Probability theory and statistics
Other domains	Graph theory, linear optimization and history of mathematics
Pedagogy	Teaching mathematics, strategies, subject specific pedagogy

Table 1: Domains of Knowledge Basis Mathematics Bachelor

The subdomain history of mathematics consists of eight learning outcomes, for instance "The teacher can give examples of the development of mathematics in relation to cultural and historical contexts". History of mathematics is also mentioned explicitly in the last domain of KBM2, which deals with pedagogy. "The teacher can use history of mathematics to enrich his/her pedagogical skills". (Kennisbasis Wiskunde Bachelor, 2017, p. 19)

[1] All KBs have recently been updated. To make things simpler for the purpose of this paper, I will refer only to the

KBM1 consists of ten mathematical domains, which are extensions to the mathematical domains of KBM2. Eight domains are obligatory and two can be chosen from a list of five domains. History of mathematics is not an obligatory domain, nor on the list of five, but it can be chosen as an extra (11^{th}) domain, or can be integrated in the rest of the curriculum. For instance history of non-Euclidean geometry can be integrated in a geometry course, (Kennisbasis Wiskunde Master, 2012). History of mathematics is not explicitly mentioned in the pedagogical domain of KBM1.

The curriculum at HvA.

In the second degree mathematics teacher training program at HvA a course on history of mathematics is programmed in the first semester of the third year.[2] The course is equivalent to 3 ECTS (equivalent to most math courses in the program). Over a period of seven weeks, the students receive an overview of history of mathematics from the early ages until the beginning of the seventeenth century. There is a 100 minutes lecture once a week. Students are provided with texts (a textbook, additional articles, some primary sources), videos and exercises. To pass, students complete a written exam.

In the first degree mathematics teacher training program at HvA, a course on history of mathematics is programmed in the second semester of either the first or the second year.[3] The course is equivalent to 5 ECTS (equivalent to most math courses in the program). Students learn about the history of mathematics, starting with the seventeenth century and finishing in the early 20^{th} century. Over the course of an entire semester there are six lectures of 150 minutes each. Students are provided with texts (two textbooks, additional articles, some primary sources), videos and exercises. They are expected to perform a bit of historical research themselves on a mathematical subject of their choice. To pass, they must hand in several written assignments, including a lesson design and a research report, and give a presentation of their research for their peers.

Description of the research.

The starting point of this research was the evaluation of the courses on history of mathematics at HvA with special interest to their effects on the beliefs and teaching skills of the students. Did the course on history of mathematics affect students' views and is there some form of transfer to their classroom activities? If the students could not identify any effects, or mainly describe the personal gain of the course they took as simply historical background knowledge with little relation to their teaching, this could serve as input for redesigning the courses.

There is very little empirical data on effects of integrating history of mathematics in mathematics education (Jankvist, 2009). The amount of empirical information on the effects of this integration in teacher training is even less. This research hopes to make a small contribution to fill this gap.

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[2] Due to curricular reform, or personal choices, some of the students have taken this course in their first or second year.

[3] The order of subjects in the curriculum in fact changes per year, because it is organized in a 'carrousel system.'

Although the first part of the questionnaire is set up in a rather quantitative way, its main goal is to gain qualitative information, as input for the teacher training program and further research.

One can think of a number of ways in which information about the history of mathematics can have an effect on students, in particular student teachers. Literature on the use of history of mathematics in the context of teacher training provided me with a list of possible effects/influences, which are stated below.

a) Influence on attitude and beliefs on the nature of mathematics as a subject (Schubring et al., 2000), (Charamboulous et al., 2009)

b) Insight in the development of mathematics and its curricula (Schubring et al., 2000)

c) Acknowledgement of processes and obstacles that can occur in developing mathematics, and thereby enhancing one's own comprehension of mathematics (Schubring et al., 2000)

d) History is an inspirer of strategies of teaching (Furinghetti, 2007)

e) Learning how to use history of mathematics in their own teacher practices (Schubring et al., 2000)

f) Influence on self-efficacy (the degree to which a teacher considers himself as capable of affecting student learning) (Charamboulous et al., 2009)

These six possible effects were reformulated into six statements (in Dutch), to which the students were asked to react. Do they (partially) agree with the statement or not? Students were asked to react to the statements on a five-point Likert-scale from completely disagree (1) to completely agree (5). All statements started with the phrase "Following the course on history of mathematics has..." and then sentences were finished by one of the above (reformulated) effects, for instance "... affected my view on the nature of mathematics." Table 2 shows the complete set of (transcribed) statements. Statement A corresponds to effect (a), statement B to effect (b), etc.

Following the course on history of mathematics has...	**1**	**2**	**3**	**4**	**5**
A) affected my view on the nature of mathematics					
B) enhanced my own comprehension of certain mathematical concepts					
C) made me more aware of conceptual- or process-related obstacles that my pupils have					
D) expanded my pedagogical repertoire					
E) taught me how to use history of mathematics with my own pupils					
F) enhanced my self-efficacy as a math teacher					

Table 2: Statements in questionnaire (transcription)

In addition to reacting to these six statements, students were invited to comment on each statement separately. It was not required to comment on every statement, but in the instructions given by the researcher to the students upon filling out the questionnaire, the value of their additionally provided explanations, examples and/or information was amplified. For each statement there was a separate question added, to be able to connect the comments to the right statement and also provoke sufficient reactions. For instance, the additional question to statement A was: "Can you comment on the way in which your view has changed? How was it before and how is it now?",

The group of students of the first degree training, who took the course on history of mathematics from January until July 2017, was asked to fill out the questionnaire in September, a few weeks after the new academic year began. Almost the entire group had finished the course by then (some still had to hand in an assignment or do some revision work on the research project). Some students had left the teacher training program altogether, or simply were not present at the time the questionnaire was taken, so not all students who took the course actually participated in this research. In November and December 2017 the course on history of mathematics in second degree training took place. Most of the students who took this course filled out the questionnaire in March 2018.

The questionnaire was filled out in class, on paper, at the start of a lecture of another course that most students were expected to take. I chose not to combine filling out the questionnaire with the written test or peer presentations at the end of the course. This way the complete course, assessment included, could be taken into account by the participants, some weeks after the entire course had finished. By doing so, it was a calculated risk that the groups of students who participated in the research were not the complete groups of students that actually took the course. The students who the course but were not present at the lecture where the questionnaire was filled out, were not tracked or asked to fill out the questionnaire later in any other way.

The results were imported to a spreadsheet to calculate the mean of the reactions to the statements. The open responses to the questionnaire were qualitatively categorized. The categories used were not defined in advance, but were a work-in-progress. Some categories turned out to be useful for open responses on several statements, which show interesting relations between some of the statements. I discuss these relations in the next section.

Results.

The results of both questionnaires are presented separately, since the students took separate courses and have different backgrounds and teaching practices. I first summarize the quantitative results on the statements, and then I adress the reactions to the statements (open responses). Finally, I provide some remarks on similarities and differences between the results of the two groups.

First degree training questionnaire.

Twenty-three students filled out the questionnaire. Table 3 shows the mean value of student reactions to the six statements and also the number of students who commented on each statement.

Statement	A	B	C	D	E	F
Mean	3.35	3.35	3.22	3.61	3.57	2.70
Comments on statement	13	12	8	15	13	7

Table 3: Reaction to statements by first degree training group

The calculated means were not meant to be interpreted separately, but can be used to arrange the statements from strongest agreement to weakest agreement. The statements on pedagogy and classroom activity (D and E) resonated most strongly with these students, while the statement on self-efficacy (F) received the weakest agreement.

As for the comments, in what follows I briefly review the statements, specify the categories used to label them, and give examples of statements made by the students. Note that comments that basically sum up to "I don't have any extra information on this statement" or that did not seem to answer the question are not accounted for and were not categorized.

Comments on statement A were arranged into five separate categories. They are presented here in order of declining frequency. The majority of responding students mentioned they gained more background information (i), some started seeing mathematics more as a dynamic subject rather as a fixed set of techniques (ii), some started seeing mathematics more as a human activity (iii), some discovered more coherence within mathematics itself (iv), and finally one student realized that mathematics can be ambiguous and debated (v).

Sample student comments on statement A:

Student #M17: From abstract science to human activity.

Student #M20: I see mathematics now more as a process rather than a result (toolkit).

On statement B, most of the responses listed mathematical topics or concepts that were understood better due to the gained knowledge of their history, but some also specifically mentioned the increased insights in coherence (see also category (iv) from statement A). When asked to comment on statement C, half of the responses described in some way that their pupils should also experience mathematics as a process rather than a product, so that can be seen as a form of transfer of category (ii) to the learning process of their own pupils.

Additional remarks to statements D and E on the possible expansion of their own pedagogical repertoire and use in their own classrooms produced mostly broad topics of school mathematics such as geometry or algebra. Some students made general remarks on using history of mathematics as a way to introduce new mathematical concepts or create more variation in lesson activities.

Statement F, on the effects on self-efficacy, had not only the lowest average on the Likert-scale, but also produced the lowest number of (affirming) reactions. A number of students explicitly stated that there was no relation between their knowledge of history of mathematics and their self-efficacy as a teacher. They almost seemed offended by the suggestion. On other statements they would simply leave a blank space if they disagreed. Students who did see a positive influence formulated it in a general way, e.g., more background knowledge gives me more insights in mathematics and therefore enhances my self-confidence as a teacher. This seems equivalent to category (i) in the first statement.

Second degree training questionnaire.

For this population, seventeen students completed the questionnaire. Table 4 shows the mean of student reactions to the six statements and also the number of students who commented on each statement.

Statement	**A**	**B**	**C**	**D**	**E**	**F**
Mean	3.65	3.88	3.53	3.76	3.29	2.88
Comments on statement	13	10	9	10	9	5

Table 4: Reaction to statements by second degree training group

When statements are ordered by calculated means of the Likert-scale of agreement, statement B (on their own understanding of mathematics) appears first. This seems logical, since the mathematical knowledge of this group of students in the bachelor program is obviously much less than that of the students in the master program, so there is more room for growth. Statement F also scores lowest in this group, the same as in the first degree training group, with similar comments that explicitly deny the relation and fewest affirming comments. The statement on the use of history of mathematics in their own classrooms comes in second to last, which can be explained by the fact that this group consists of less-experienced teachers, which might be holding back on this type of activity in their teaching.

Comments on statement A were divided into five separate categories, which mostly correspond to the categories in the other group. Again the order is by declining frequency. Students mentioned they gained more background information (i), recognized, in particular, more different cultural contributions in the history of mathematics (ii), started seeing mathematics more as a dynamic subject rather as a fixed set of techniques (iii), saw more coherence within mathematics itself (iv), and made connections with the learning of their own students (v). The new category (ii) seems appropriate, since this course explicitly pays attention to the contribution of non-Western cultures to the development of mathematics.

Statements B and C were commented on in a rather similar way. Almost all comments contained examples of specific topics or concepts of school mathematics. Amongst those topics the concept of number (e.g., negative numbers, fractions, square roots, and zero) was mentioned frequently, as well as solving linear and quadratic equations.

Sample student comments on statement D:

> Student #B3: For instance to visualize equations with geometry.
>
> Student #B14: Introducing variables with “The thing plus the root of the thing.”

Although statement D scored rather high quantitatively and the comments on this statement produced plenty of concrete examples (like the ones on statement B and C), the use of history of mathematics in students' own classrooms (statement E) mostly resulted in general ways of using history of mathematics, such as introduction or variation.

Overal remarks.

It was surprising to see students describe effects that taking the course on history of mathematics had on them with such detail, especially in the first degree training group. Without any concrete examples mentioned by the researcher, students were able to mention words like coherence, human activity, and ambiguity. This can indicate advanced personal reflection skills on the part of students, or this may have been provoked successfully by the design and implementation of the course.

Some of the differences in reactions between the two groups seem naturally connected to the level of their knowledge of mathematics and teaching experience. Students in the second degree training group were better at specifying the relation to the topics in school mathematics, which seems logical considering the contents of their course. Early developments of mathematics can actually be found in the curriculum for 12- to 15-year-olds, until the coordinate system of Viète and Descartes in the early 17th century, but when we discuss Weierstrass and Cantor with students in the first degree training, this is a much more distant to the concepts they teach in their own classrooms.

Both groups were rather firm in their rejection to influence on their self-efficacy. One could argue that the gained knowledge, both mathematically and pedagogically, should be rather closely connected to the confidence of a teacher, but for the students who took these courses and completed the questionnaire this was a bridge too far.

Discussion.

The obvious point of discussion here is the design of the courses. This was not aligned with the six effects of the use of history of mathematics in the context of teacher training which were obtained from literature. Further research is necessary to focus on one or more effects, which must be attended to beforehand in the design of the courses. In particular, the way the courses are assessed should be taken into account. Still the results of this research give us valuable information on what effects take place, by courses designed in the described manner, at least as reflected by the students themselves. This leads to another point of discussion.

It is important to note that the results are purely based on self-reported opinions of the students themselves, on their self-assessment of their knowledge, and their views 'at the desk.' That means: this is what they think of themselves, their views and skills, outside the classroom. To get a better picture of the effects there should be some form of 'in action' research in their classrooms.

The results of this pilot empirical study indicate an added value of knowledge of history of mathematics for teachers. The results can be used as input for the design of courses and other activities involving history of mathematics in teacher training. They may also be useful for further research on the partnership between mathematics, history and education.

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