Practitioner's Corner: Episode 2

History of Geometry

Author(s)
van den Bogaart, Desiree

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PRACTITIONER'S CORNER

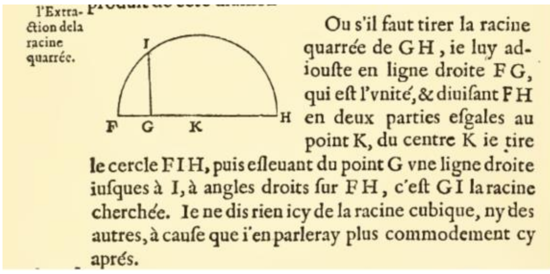
Episode 2

History of Geometry

This is the second posting in a series in which mathematics teacher educators share experiences with teaching history of mathematics, within the context of teacher education for both lower (aged 12–15) and upper secondary levels (aged 16–18). Let me start by briefly introducing myself and the context I work in. I am a mathematics teacher educator at the Amsterdam University of Applied Sciences. I've been working at this university for the past eight years, after working as a mathematics teacher in secondary school for ten years. I teach a variety of classes on mathematics, but also on teaching methodology and professional development as a teacher.

At my university, the amount of time I have with my students is limited to seven 90-minute sessions in a single semester. In my pre-service lower secondary level teacher group we rapidly go through the history of mathematics up until Descartes' analytical geometry and in my in-service upper secondary level teacher group I address the history of mathematics roughly from the emergence of calculus in the seventeenth century to the Millennium Problems in the year 2000. In this post, I describe some activities on the history of geometry. They use primary sources, GeoGebra, and end up with cat litter.

In the first session of history of mathematics to my in-service upper secondary group, I try to bridge the gap between early history and modern history. We focus on the seventeenth century and about half of the session is spent on Descartes. My students read some of his *Discours de la Méthode* (in the original language) and try to make sense of his geometric form of multiplication, division and taking the square root.

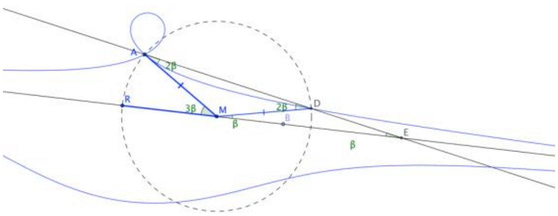

l'Extraction dela racine quarrée.

Ou s'il faut tirer la racine quarrée de GH, ie luy adiouste en ligne droite FG, qui est l'vnité, & diuisant FH en deux parties esgales au point K, du centre K ie tire le cercle FIH, puis esleuant du point G vne ligne droite iusques à I, à angles droits sur FH, c'est GI la racine cherchée. Ie ne dis rien icy de la racine cubique, ny des autres, à cause que i'en parleray plus commodement cy aprés.

To test if my students really understand the constructions that Descartes is proposing, I ask them to hand in a GeoGebra file in which they show how Descartes constructs $\sqrt{a^2+b}$, with a and b of variable lengths (parameters). It is relatively easy to understand the proof of the different parts of this construction, using similarity as a main geometric argument, but to program GeoGebra performing this procedure in full requires that little bit extra. As a teacher, I can check fairly quickly if they did it correctly.

After discussing the principles of Descartes shifting between geometry and algebra, we turn to the *conchoid of Nicomedes*. The intensive study of curves is also a typical feature of seventeenth century mathematics. And although Descartes was clearly not the first to study the conchoid, it fits rather naturally in this session. Most of the students haven't heard of this curve before and when they have, it can be captured by "some curve that has something to do with trisecting an angle." If you need more information on the conchoid, see for instance https://en.wikipedia.org/wiki/Conchoid_(mathematics)

I have taught this class a couple of times and I've noticed my students find it difficult to grasp the construction of the conchoid. One thing that helps them to understand the conchoid is the proof of the trisection of the angle. Once my students have seen through different similar triangles that the angle you end up with at the intersection with the curve is actually one third of the angle you start constructing the conchoid with, they can understand the meaning of several parts of the construction better.



But, to understand the proof, one does need to understand the construction. I use (again) GeoGebra to visualize it, but I have noticed with my students that other tools for demonstrating how the construction works have way more impact in this case. I use two alternatives for GeoGebra.

First I use a wood and iron construction made by a colleague from another university (Figure 1). With a large piece of cardboard and some markers, the curve is partially drawn in front of the class. I usually let some students assist me while I'm doing this. I show the wooden plate with trench and metal attachments and build the construction piece by piece, until finally a student draws one part of the conchoid on the piece of cardboard.

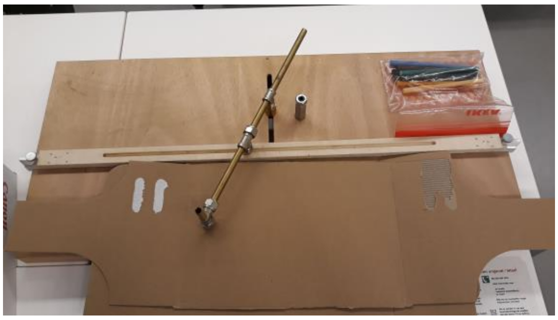


Figure 1. Wood and iron construction to draw the conchoid.

But the second visualization is much more spectacular, and much more fun. I have the students form a *human conchoid*. When we (part of the credit for the human conchoid goes to my husband Theo, who also happens to be a mathematics teacher educator with an interest in history) came up with this idea, it was late in the evening. All the materials I needed for the actual construction, such as a large piece of electricity pipe and a connection piece, could easily be attained at our local home improvement store. But how was I going to leave a visual trace on the floor, that could also be easily cleaned up afterwards? Colleagues who have subsequently done the human conchoid themselves in their classes have informed me they did it outside, on sandy ground, so they could use water to leave the trace. But I was supposed to do this mid-winter inside a school building, so water was not an option. After experimenting with both sugar and rice (both too small to leave a good trace; it spread all over the place) I desperately searched my house for alternatives and ended up in the basement were we keep the (clean) cat litter. The grains were the perfect size. An empty soda bottle filled with these grains was the final part of the construction.

When I present the human conchoid to my students, I ask for volunteers, based on human characteristics. ***Line l*** (see Figure 2) is drawn on the floor with tape. The first volunteer needs to be a very stubborn person, one who will not move an inch once they have chosen their position. This person has to stand still the whole time, at a small distance from **the tape**, and serves as the origin ***O*** of the conchoid. They will have to guide a piece of electric pipe (*line m*), conducted by volunteer ***P***. In the search for person ***P***, I ask for someone who has "walked the line" successfully. ***P*** has to walk carefully down the taped ***line l***, while holding *line m* (piece of pipe) on a fixed position in the middle. To be precise: ***O*** doesn't hold *m* at a fixed spot, but lets it slide down his/her hands (using a piece of pipe with a slightly larger diameter).

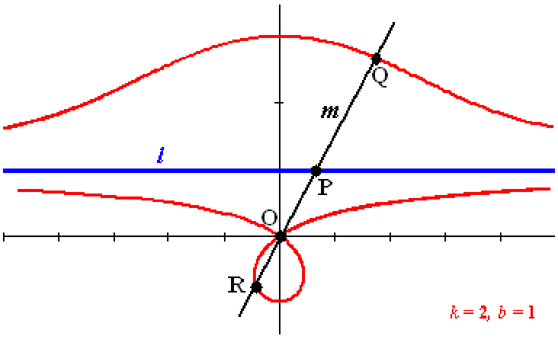


Figure 2. Image to accompany the construction for the human conchoid.

Thirdly, ***Q*** has to be a very flexible person. He/she holds the soda bottle with grains in one hand, and therefore makes *the trace*. But ***Q*** can't move independently. ***Q*** has to hold in the other hand the physical *line m* at a fixed point, while the line is being moved by ***P***. So ***Q*** keeps the line at a fixed position, but has to correct their own position while ***P*** walks along ***line l***. And then *the upper curve of the conchoid* takes shape, in cat litter, on the floor.

According to the famous anecdote, Descartes was lying in his bed and saw a fly on the ceiling, which led him to develop his coordinate system. With this contemporary teaching discovery, it was actually a cat that led to the development of the human conchoid.

Desiree van den Bogaart
Amsterdam University of Applied Sciences (The Netherlands)
d.a.van.den.bogaart@hva.nl