Practitioner's Corner

Episode 1

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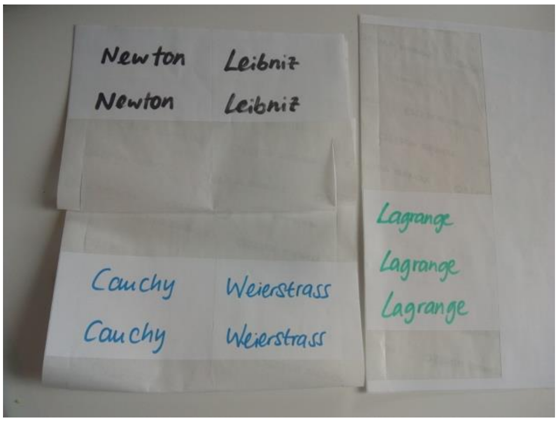
Episode 1

Let me start with shortly introducing myself and the context I work in. I'm a mathematics teacher educator at the University of Applied Sciences in Amsterdam (The Netherlands). I've been working at the university for the past eight years, after working as a mathematics teacher in secondary school for ten years. I teach a variety of classes on mathematics, but also on pedagogy and professional development as a teacher. In this series I will focus on my experiences with teaching history of mathematics, within the context of teacher education for both lower and upper secondary levels.

At my university, the amount of time I have with my students is limited to seven 90-minutes sessions in a single semester. In my preservice lower secondary level teacher group we rapidly go through the history of mathematics up until Descartes' analytical geometry and in my in-service upper secondary level teacher group I have to address the history of mathematics roughly from the emergence of calculus in the seventeenth century to the Millennium Problems in the year 2000. In this episode I will describe two different activities: one from a session on the history of calculus and one session on Euler.

Session 1: The history of calculus

When my students entered the classroom, I handed out nametags in random order (tags are shown on photo on the left). The students adjusted their tag to their shirt and took a seat. All students tagged Newton formed a group, so did the Leibnizs, Lagranges, Cauchys and Weierstrass.



I handed them a single translated piece of mathematics, originally written by their namesake, on the derivative of $y = x^p$ and asked them to closely read it and discuss it in their groups.

Newton
De *fluents* x en y zijn gerelateerd volgens de vergelijking $0 = y - x^p$. Het moment van de fluent x is het product $\dot{x}o$ van de *fluxion* $\dot{x}$ en een oneindig kleine hoeveelheid o. Net zo is het moment van de fluent y gelijk aan $\dot{y}o$. Aangezien de momenten (zeg $\dot{x}o$ en $\dot{y}o$) van fluents (zeg x en y) de oneindig kleine toevoegingen zijn waarmee de hoeveelheden toenemen gedurende een oneindig kleine tijd, zal de fluent x en y na een oneindig kleine tijd gelijk zijn aan $x + \dot{x}o$ en $y + \dot{y}o$. Aangezien $0 = y - x^p$ geldig is *op alle tijdstippen*, geldt ook $0 = (y + \dot{y}o) - (x + \dot{x}o)^p = y + \dot{y}o - x^p - px^{p-1}\dot{x}o - \frac{p(p-1)}{2!}x^{p-2}\dot{x}^2o^2 - \cdots.$ Nu geldt volgens aanname $y - x^p = 0$ en als je deze term weghaalt en vervolgens deelt door o, krijg je $0 = \dot{y} - px^{p-1}\dot{x} - \frac{p(p-1)}{2!}x^{p-2}\dot{y}^2o - \cdots.$ Maar aangezien o oneindig klein is, zijn termen die dit als een factor hebben gelijk aan niets in vergelijking tot de andere. Daarom haal ik ze weg en houd ik over: $0 = \dot{y} - px^{p-1}\dot{x}.$
Leibniz
Neem een infinitesimale verandering dx van x. De infinitesimale toename dy van y is dan $dy = (x + dx)^p - x^p = px^{p-1}dx + \frac{p(p-1)}{2!}x^{p-2}(dx)^2 + \cdots.$ Maar aangezien $(dx)^2$ en hogere machten infinitesimaal veel kleiner zijn dan dx, geldt: $dy = px^{p-1}dx.$
Lagrange
De afgeleide $f'(x)$ van een functie f in x is *per definitie* de coëfficiënt B in de Taylor-ontwikkeling van f rond x: $f(z) = A + B(z - x) + C(z - x)^2 + \cdots.$ Schrijf $y = f(x) = x^p$.   Dan geldt: $f(z) = \left(x + (z - x)\right)^p = x^p + px^{p-1}(z - x) + \binom{p}{2}x^{p-2}(z - x)^2 + \cdots$ en dus $f'(x) = px^{p-1}$.
Cauchy
De afgeleidefunctie van f is de limiet van $[f(x + i) - f(x)]/i$ als i tot nul nadert. In dit geval geldt: $\frac{f(x+i)-f(x)}{i} = px^{p-1} + \binom{p}{2}x^{p-2}i + \cdots.$ Door i nu steeds dichter bij 0 te nemen, kan het rechterlid van deze vergelijking zo dicht bij px^{p-1} komen als je wenst. Daarom geldt $f'(x) = px^{p-1}$.
Weierstrass
Per definitie: $f'(x) = \lim_{\delta \to 0} \frac{f(x+\delta)-f(x)}{\delta}.$ Om nu aan te tonen dat als $f(x) = x^p$ dat dan $f'(x) = px^{p-1}$ gaat men als volgt te werk: Gegeven x. Gegeven $0 < \varepsilon < 1$. Zij $\delta < \frac{\varepsilon}{p2^pX}$ met $X = \max(1, \lvert x^p \rvert)$. Er geldt: $\left\lvert \frac{f(x+\delta)-f(x)}{\delta} - px^{p-1} \right\rvert = \left\lvert \sum_{i=2}^{p} \binom{p}{i} x^{p-i}\delta^{i-1} \right\rvert$ *[binomium]* $\leq \sum_{i=2}^{p} \binom{p}{i} \lvert x^{p-i} \rvert \delta^{i-1}$ *[driehoeksongelijkheid]* $< \varepsilon,$ *[iedere sommand is kleiner dan $\frac{\varepsilon}{p}$].*

After about twenty minutes, the students split up and formed new groups, in which all of the historical figures and sources were represented. They shared their findings on their own source and then discussed similarities and differences, tried to see a development through time of the concept of a derivative and its notation. (This took about 40 minutes.) The activity ended with a class discussion. The names of the mathematicians were shown on a digital "wheel of fortune." Using this tool, I selected students to report their findings to the whole group.

Jigsaw technique

This type of classroom activity is called "expert-method" in Dutch terminology, or "jigsaw technique" in American educational literature. It relies on dependability between students. It breaks classes into groups and breaks assignments into pieces that the group assembles to complete the (jigsaw) puzzle. The Dutch title refers to the effect that a student becomes an expert on a specific piece of theory or part of the assignment.

I use this structure often in my history of mathematics classes, because I believe it fits the contents in a very natural way. It can be used to let students identify with an actual expert from the past, such as Newton or Leibniz, and they become that expert in a discussion. The students become the historical figure, who was an expert in their own time and obviously is an expert on his own written material. Roleplaying is a strong gaming mechanic. This way they engage more deeply in the process of doing mathematics, finding arguments, debating their work with other students. The roleplaying has a natural connection with the use of primary sources. The primary sources are actual artifacts to the character and add to the historical atmosphere. On the use of primary sources, much has been written, for instance, by Janet Barnett and colleagues.

The jigsaw-technique or expert-method can be used in a regular mathematics class as well, as long as the subject can be divided into several distinct pieces or tasks. For instance when you discuss geometrical shapes, probability distributions, or different ways of solving an equation. The next example is also from my history of mathematics class, but here the different pieces of the puzzle are not mathematicians, but pieces of mathematics.

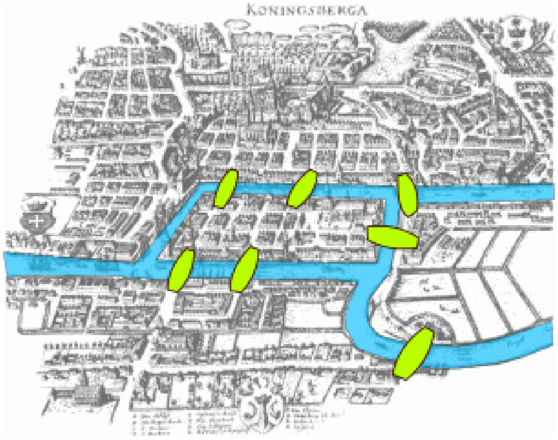
Session 2: The expertise of Euler

When I prepared my lecture on Euler for the first time, I immediately knew I wanted to use the expert-method on this one as well. The number of different types of mathematical activities that Euler did, on such a groundbreaking level, still amazes me. He was an expert on just about everything in mathematics in his time and I wanted my students to get a glance of this great achievement.

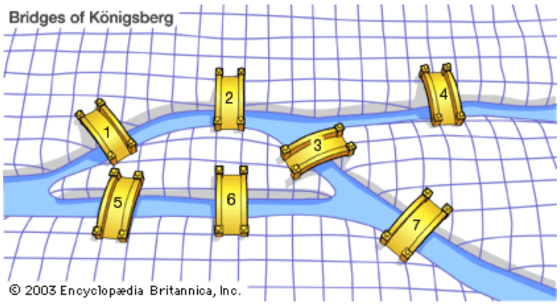
I came across Ed Sandifer's brilliant archive, "*How Euler did it*." This made things rather easy for me. I selected seven of the columns from the archive. I didn't want to include the Königsberg bridge problem, because it is rather easy and too well known. But I also felt I couldn't leave it out, just in case someone would have missed that class or forgotten it was Euler who starred in it. So, I used the map of Königsberg as a tool in the second part of jigsaw format, as I will explain further on.
The class was divided into seven equal groups. Each group was given a different

number from 1 to 7 and was handed several copies of one of Sandifer's columns and some additional sources (e.g., a book or URL). For instance, the first group discussed a text on Euler's contribution to Venn diagrams, the second group did $e^{i\pi} + 1 = 0$, another group did his formula $V + F = E + 2$, one group discussed his work on amicable numbers, one did the St. Petersburg paradox, etc. I asked the students to study the text on Euler's work, and all prepare a poster sheet on an A3 sheet of paper, in order to be able to present this work to others later. The groups had about 30 minutes to create their poster.

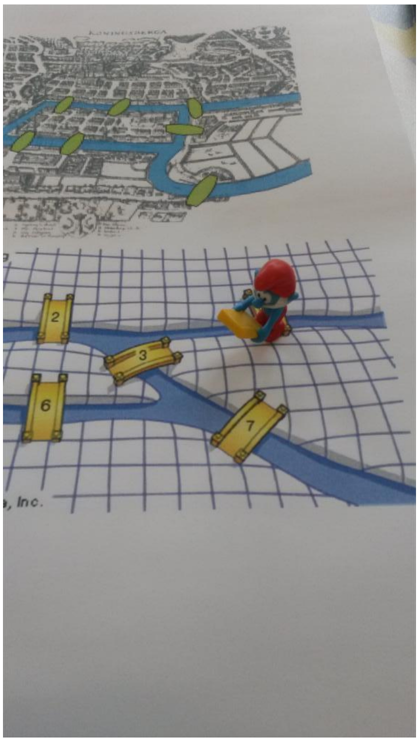
The students then had to form new groups, in which all numbers 1 to 7 were represented. Each newly formed group was handed a placemat with two maps of Königsberg, with all the bridges on it, as shown below. One map was a somewhat authentic drawing of the city, the other was much more schematic and contained all bridges numbered from one to seven. I also handed them a toy figure that represented Euler. (I used Big Smurf for this, since at the time there was a collect-all-five action at our local groceries store. And then, of course, Big Smurf has a beard and carries a book. You can use a regular game pawn for this as well, but I believe the personalization of Euler on the map adds to the full experience.) Big Smurf/ Euler had to walk around through the city of Königsberg and each time he crossed a bridge the group member with the same number had to present his or her poster with Euler's work on it. Each bridge could only be crossed once. So obviously this meant that one group member would not be able to present their topic. These students were asked to present their work to the whole group at the end of the session.



Map 1: Ancient city of Königsberg



Map 2: Schematic version of the bridges


Big Smurf (aka Euler) in Königsberg

Final remarks

I have tried to share some classroom activities and design principles from my experience as a history of mathematics teacher within a teacher training program. I believe several aspects of these sessions can be used in different contexts, with different contents and materials. If you wish to comment, share your own experiences, or have any further questions, please feel free to contact me.

Resources

On the jigsaw-technique: www.jigsaw.org
On role-playing games:
http://aestranger.com/role-playing-games-educational-tools/
The wheel of fortune:
https://wheeldecide.com/wheels/games/wheel-fortune-spinner/
Ed Sandifer's "How Euler did it":
http://eulerarchive.maa.org/hedi/

On use of primary sources:

Barnett, J. H., Pengelley, D., & Lodder, J. (2014). The pedagogy of primary historical sources in mathematics: Classroom practice meets theoretical frameworks. *Science & Education*, *23*(1), 7-27.

On Dutch teacher training:

Bogaart-Agterberg, D. van den (2019) *History of mathematics in Dutch teacher training.* Paper to be published in the proceedings of the 11th Congress of European Research in Mathematics Education, Utrecht, Netherlands.

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PETER RANSOM, THE MAN KNOWN TO TRAVEL WITH A SWORD AND CANNON BALLS TO MANY PARTS OF THE GLOBE

On the occasion of the Queen's birthday in 2019, **Peter Howard Ransom** has been appointed a Member of the Order of the British Empire (MBE). This is a great honor for him, his wife Janet, daughters Madeleine and Claire, and grandsons. And it is also an acknowledgment for HPM community and, in general, of mathematics educators. As a matter of fact, the motivation for the appointment is "For voluntary service to Mathematics Education." The following notes on Peter's career illustrate his active role in the community of mathematics educators.

He started teaching mathematics in state schools in September 1977 to students aged 11 to 18 years old. After teaching in three schools he was then employed in 1994 by SMP, the Schools Mathematics Project, to write textbooks for the age range 11 to 16. When that work was completed in 1998, he returned to teaching 11- to 16-year-olds in state secondary schools. While at The Mountbatten School in Romsey, he edited the HPM Newsletter from 2000–2004, supported by his school which distributed the newsletter. He was delighted to receive notification of the MBE, with the citation "For voluntary service to mathematics education."

He has contributed to many mathematical communities in the UK (MA, IMA, BSHM, LMS, ATM, UKMT, Ri and Secretary for