Why digital tools may (not) help by learning about graphs in dynamics events?

Author(s)
Palha, Sonia

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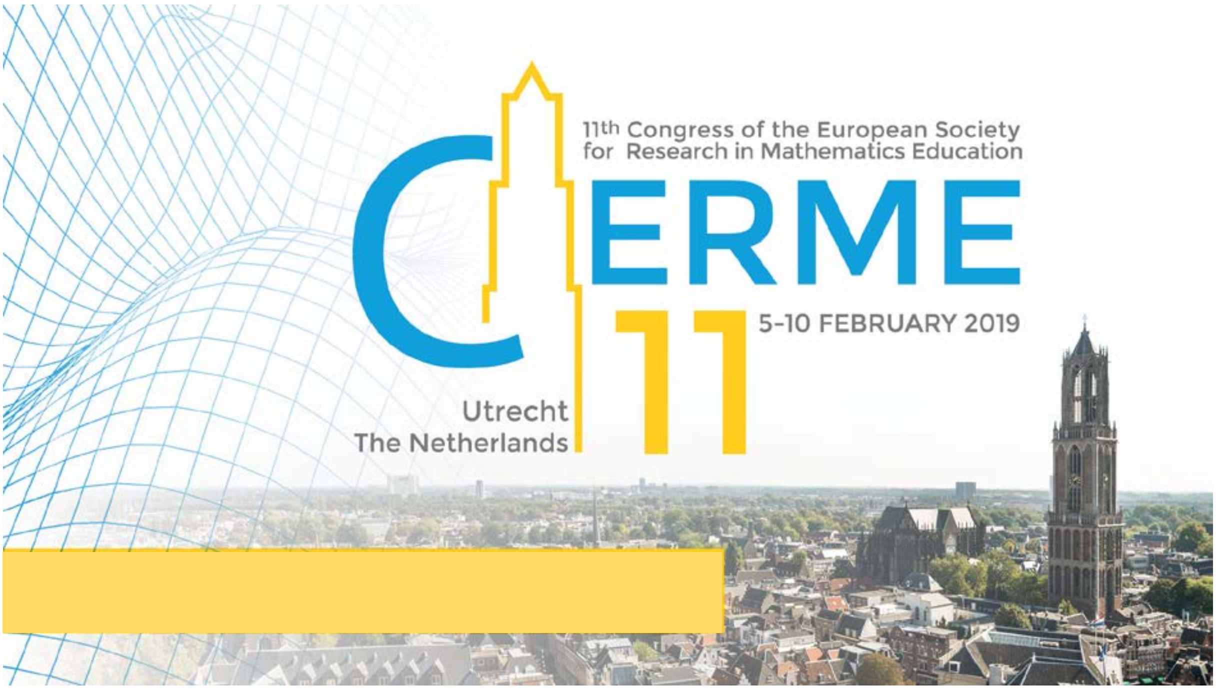

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Why digital tools may (not) help by learning about graphs in dynamics events?

Sonia Palha

Amsterdam University of Applied Sciences, Centre for Applied Research on Education (CARE), Amsterdam, The Netherlands; s.abrantes.garcez.palha@hva.nl

Understanding graphs representing dynamic events is a challenge for many students at all levels. And technological tools can provide support in overcoming some of these difficulties. In our research we developed a digital tool that enables students to create, modify and improve graphs from dynamic events using interactive animations and intrinsic feedback. In order to get insight about why the tool helped (or not), the students we conducted a qualitative study in which we interviewed nine students who used the tool. The results offer insight in students' learning and thinking about dynamic graphs and how digital feedback can afford that. These results are useful for researchers, developers and teachers.

Keywords: visualization, intrinsic feedback, graphs, interactive tool, learning with technology.

Introduction

Dynamic graphs remain an essential subject at all levels of High School in mathematics, but it remains a challenging topic for students and even for teachers (Carlson, Larsen, & Lesh, 2003; Moore & Carlson, 2012). Reasons behind students' difficulties frequently involves problems in visualizing change and variation, limited understanding of functions and co-variational reasoning. Moreover the construction of a global graph from a realistic situation and stretch is similar to mathematical problems solving and requires mathematical thinking (Moore & Carlson, 2012; Thompson, 2011). Digital tools can support students to deal with these difficulties in several ways. Dynamic software as Geogebra and applets can support students in visualizing relations through enabling them to draw, move and modify graphs within different representations. And interactive applications that connects animations and graphs can be used to explore relationships between phenomena, and it's graphical representation. The learning potential of dynamic tools can be highly improved by including possibilities for students interaction with the tool like students' own productions and incorporated feedback-features (Laurillard, 2013). However, this also put more demands on the tools' design especially when they build upon students free-hand productions. More knowledge and research on students learning with this type of tools are needed to inform tool-developers and teachers.

Interactive Virtual Math (IVM) is an example of such a tool. It generates and builds upon students free-hand graphs, and it has incorporated feedback. The tool was developed through developmental research (Palha & Koopman, 2016) with the aim to improve students' learning about graphs by dynamic events. The tool creates opportunities for students to experience the thinking and reasoning that is needed to generate, revise and modulate a graph from themselves. When entering the tool they get tasks that encourages them to imagine two variables changing simultaneously. The tool requests the students to produce the graphical representation and the verbal explanation for this relation, which requires students to represent their concept image graphically and verbally. With the help of the incorporated feedback, the student is challenged to think, reason and act upon his own construction. This is an innovative pedagogical feature of the tool that requires deep knowledge to be developed properly. The tool also includes the use of Virtual Reality (sound, movement, interaction), which is however very limited, but it is expected to improve the experience of the graphic situation.

Previous research (Palha, 2017) about IVM shows that students (age 13-17) find the tool useful because it assists them to improve graphs and/or to gain a more thorough understanding of the subject. The study was a teaching experiment involving three classes at secondary and one class at tertiary education in The Netherlands that used IVM during one lesson (45-50 minutes). Seventy-nine students reported through questionnaires about their experience with the tool and what supported them the most. However, this data didn't provide us much in depth knowledge about the way students improved their graphs and why. During the experiment we also interviewed nine students, and we collected their backlog files in the tool. This data has been analyzed recently, and it provides us with new insights on learning with the tool.

In this paper we report the results of these qualitative analyses. We use the framework for covariational reasoning of Carlson, Oehrtman, & Engelke (2010) and the notion of intrinsic feedback (Laurillard, 2013) to interpret and evaluate the way students utilized the tool. The guiding research question is: how does the tool enable students to improve their graphical representation and/or understanding about dynamic events?

Theoretical background

Learning about dynamic graphs

An example of a dynamic event is the following situation: imagine a bottle filling with water. Sketch a graph of the water's height in the bottle (Carlson et al., 2010). To solve the bottle-task, the students will need considering how the dependent variable (height) changes while imagining changes in the independent variable (volume). The coordination of such changes requires the ability to represent and discern relevant features in the shape of the graph. These mental actions are in the core of covariational reasoning and are clearly defined in the framework of Carlson et al., (2010). The authors define covariational reasoning as entailing five mental actions: (M1) coordinating the value of one quantity with changes in the other; (M2) coordinating the direction of the change; (M3) coordinating the amount of change of one quantity while imagining successive changes in the other quantity; (M4) coordinating the average rate of change of the function with uniform increments of change in the input variable; (M5) coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function.

Several researches (Thompson, 2011; Saldanha & Thompson, 1998; Castillo-Garsow, Johnson, & Moore, 2013) reported students' difficulties in engaging with the mental activities M4-M5. These students also have difficulty explaining why a curve is smooth and what is conveyed by an inflection point on a graph. According to Carlson et al., (2010). Students should have opportunities to experience the covariational nature of functions by dynamic events. Thompson (2011) too states it is critical that students first engage in the mental activity to visualize a situation and construct relevant quantitative relationships prior to determining formulas or graphs. Ellis (2007) suggests that learners should be helped to focus on quantities and generalizations about relationships, connections between situations and dynamic phenomena.

Intrinsic feedback

Technological tools can be designed to help to create a learning environment that fits the characteristics mentioned in the previous section. The learning environment that we have in mind involves learning from experience and by reflection on ones' own productions. These are underlying ideas in theories that account learning as an active and social process. With the use of specific type of tasks, students can translate their concepts into practice. They can compare and evaluate how well they achieve some learning goal. And employ this to improve their initial concept and practice and develop knowledge (Laurillard, 2013).

Digital tools with capability to provide intrinsic feedback can assist students with learning because they: (i) enable the students to modulate their concept image and generate actions that brings them nearby a goal and; (ii) use the feedback of the tool to modulate their practice and revise their actions. According to the framework of Laurillard learning occurs when students engage in a successive cycle involving these actions. Actions and feedback drive the internal modulate-generate cycle that links the students' conception to their repertoire of actions as practice. Through reflective observation students can modulate the abstract concept in the light of concrete experience and generate new actions as active experimentation (p.168).

The prototypical tool IVM aims at creating a practice environment for the learning about graphs by dynamic events, in which student engage in the internal successive cycles described above. Moreover, the tool incorporates features designed to engage students in modulate-generate-revise actions that foster the development of covariational reasoning Carlson et al. (2010). With the tool there is a sequence of trials, mostly with a cyclic character that need to occur and be repeated to assure learning (see table 1 with the description of the tool).

Self-construction -task	
Task	the tool sets a task to construct a graph from a non-trivial dynamic event
Learner	the learner draws a graph by the dynamic event and explains his/her thoughts
Compare -task	
Task	The tool displays a second task with a dynamic event that corresponds to the more common mistake made by learners with regard to this type of task
Learner	The learner uses the comparison to draw (if needed) an improved graph and/or adapt the explanation
Task	The tool offers a menu in which students can choose between seeing help or to submit their graph
Help 3D-animation (optional)	
Learner	The learner chooses help 3D-animation
Task	The tool displays an animated version of the dynamical event in which the same amount of water is added each time student presses a button
Learner	The learner notices that the height of the water increases in the jar and that, for the same amount of added water, the incremented heights are different and depending on the form of the jar; the learner reflects on these relations and becomes aware that the increment of the height is smaller when the jar is broader -in the middle and larger in the extremities. The learner uses the visualization to draw (if needed) an improved graph and/or adapts the explanation
Help interactive animation (optional)	

Learner	The learner choose help interactive animation
Task	The tool displays an animation of the dynamical event and an empty cartesian graph in which the learner can move given dots to the estimated height every time the same amount of water is added; both actions, adding the water by pressing a button and moving the dot are performed by the learner.
Learner	The learner moves vertically the first dot to the position he/she thinks that the height of the water will reach and then press the button. The water fills in the jar at a certain height. The learner compares the estimated and the reached height and uses the comparison to move the dot to a more precise location.
Feeback at the end- reward feature	
Learner	The learner submits his/her solution
Task	The tool displays the figure of the jar corresponding to the learner drawings
Learner	The learner compares the form of the jar with the initial form and reflects on the differences and similarities and on the relation between the graph and the shape of the jar. The learner can go again through the tool and use the visualization to draw an improved graph and/or adapt the explanation

Table 1: description of the tool Interactive Virtual Math (IVM)

Method

We interviewed nine students from 10th and 11th grade who utilized the tool in the classroom: two boys (Niels and Abel) and two girls (Miriam and Jenny) from school A, 11th grade, 16-17 years-old; two girls (Olivia and Anne) from school B, 10th grade, 15-16 years-old and two boys (Gerry and Arion) and one girl (Manon) from school C, 10th grade 16 years old. In schools A and B students utilized a computer and construct the graph with the mouse. In the school C school students used a Iphone and draw with the finger. The interviews were semi-structured and focus on the way students used the tool and the reasons to modify or not the original graph. The interviews were performed by the author of this paper or by the teacher. They took about 5-10 minutes per student and were recorded in audio. Additional data included the results of a survey conducted during the lesson and the students productions registered in the tool self (backlog of the tool). The data analysis was qualitative and based on the covariational framework van Carlson et al. (2010) and the notion van learning cycles from Laurillard (2013).

Findings

Students transformed an incorrect graph into a correct graph with the tool (n=5)

To draw a correct graph students need to coordinate the average rate of change of the height as function of the volume and imagining it changing instantaneously with continuous changes in the independent variable (levels MA4 and M5 of the covariation framework). Analysis of the backlog of the tool showed that five from the nine students generated a correct graph, but only after a second or third trial. In their first trial these students sometimes draw straight lines instead of curves or draw a concave up-down graph instead of a concave down-up or an inaccurate global shape. This suggests that the students could reason at the levels MA2 or MA3 of the covariation framework at the start of the assignment. In the interview students were asked to explain how they transformed their graph and why they transformed it. Table 2 provides an overview of the way students engaged in the learning cycle and generated, modulated and revised their initial graph with the tool.

Students	How and why the graph was transformed and which tool-features enabled this
Jenny	Jenny *modulated her concept image* and *generated a better-shape grap*h after two trials. The student used the feedback from the reward-feature to *revise* and *modulate the steepness* of the curves.
Niek	Niek *modulated his concept image* in two trials. The student used the feedback from the reward-feature to *revise* and *modulate* a concave up followed by a concave down graph into a correct graph.
Abel	Abel *modulated* his concept in two trials. The student used the feedback from the reward-feature to transform part of the graph (an increasing straight line) into a concave down curve.
Gerry	Gerry *modulated* his concept in three trials. The student used feedback from the reward-feature and help 3D-animation to *revise* and *modulate* three increasing straight lines into a concave down followed by a concave up curve and to improve the shape of the graph.
Manon	Self-construction of the graphs (with no formula) enabled Manon to reflect before drawing the graph and therefore *modulate her concept image*; feedback from the reward-feature enabled her to *generate and revise* actions as she improved her graph. Both help-features were consulted but not used.

Table 2: the tool enables students to produce a correct graph with the tool (n=5)

All five students realized that their graph was not correct because of the feedback from the reward-feature. One student, Niels explained:

> "when I saw the feedback at the end (....) you can work towards a nice graph and you know how it really should look like, because you know that at a certain point will be slower than elsewhere (...) " when your vase is inside you know that then you have to go to the other side (...) so first slowly and then faster instead of first faster and then slower "

Manon found the tool very instructive especially because the self-construction- task that challenged her to generate an action from her practice repertoire and reflect on it:

> “'I was already ready to put something in the Graphic Calculator or to do something like that. But this was not possible in this case and I really had to think: ok, what is happening here? what happens in the middle? what happens there ... and that I found very instructive"

Also Gerry, mentioned the self-construction task as a useful feature to visualize the graph.

> "You start without having an idea of what looks like; you try to imagine yourself and then when you see the film you realize that it is very different from what you had thought up”

Gerry felt also helped by help 3D-animation and he explained why:

> "you saw in the animation (3D) the round shape of the bowl and you saw better how the water was distributed. In the beginning that a lot more space had been taken and in the middle there is much wider and because of that the graphs were very different "

Further, all the students with exception of Abel considered that because of the tool they understand the subject better; Abel considered that he already understood the subject.

Students realized they had an incorrect graph and did not succeed in transforming their graph into a correct one with the tool (n=3)

Three of the nine students (Olivia, Anne and Arion) have done two, sometimes three trials with the tool but were unsuccessful to generate a correct graph. Analysis of the backlog of the tool showed that the students initially generated a graph with one or more straight lines, which corresponds to M2 in the covariational framework. Interviews with the students provided us with more insight about how students revised and modulated their graphs (Table 2).

Students	How the graph was transformed and which tool-features enabled this
Olivia	The first trial is an increasing straight line, the second trial is a concave down line and third trial is a concave up. The comparison-task and the feedback from the reward-feature enabled student to *modulate* her concept in the sense that the lines should not be straight. Olivia *generates* and *revise* actions when she produces new graphs with curves. She *revised* and *modulate* her graph but *she could not generate* a correct graph
Anne	In the first and second trial the graph is an increasing straight line and the slope of the second trial is smaller. The feedback from the reward-feature and help-3D animation enabled Anne to *(partially) modulate* her concept as she realized that the lines should not be straight. She *revised* and *modulate* her graph but *she could not generate* a correct graph.
Arion	In the first and second trials the graph has three increasing straight lines. The feedback from the reward-feature enabled student to *modulate* his concept: that the graph should not have straight lines. However, for technical reasons the student could not work with the tool and use the feedback to improve his graph.

Table 3: the tool enables students to realize their mistake but not to improve it (n=3)

Olivia considered that the tool helped her to improve the graph and she explained why:

> "it really helped that in the end I converted my drawing into that shape, as a sphere because I saw a bit connection" (…) "If there is no straight line then there must be a concave up line or a concave down line – I will try both.

Olivia didn't used the help-features because she doesn't think that it would be useful to achieve her goal of drawing a correct graph. The visualization of the context is not the problem according to the student but the fact that she can't draw the graphical representation:

> "'I went very often trying the tool because I was curious (….) with the movies I do not know much what I had to show"

The other two students did not consider that the tool helped them to improve the graph. Anne because the straight line remained a straight line and Arion because the tool didn't work well and he could not keep trying (there were in some smartphone technical problems). For these students the intrinsic feedback provided by the tool enabled them to engage in the modulate-generate cycle and modulate their initial concept image towards a more sophisticated one (although not yet correct).

Students realized they had an incorrect graph but did not do nothing about it (n=1)

Miriam was the only student who produces an incorrect graph and did not invest effort to improve it with the tool. In her initial trial she constructs an incorrect graph with three straight increasing lines, which corresponds to the mental action M3 in the framework covariational reasoning. She didn't consult the help-features. In a second trial Miriam worked with a peer-student who produced a similar graph to her but with curves instead of straight lines. Noticing the difference enabled the student to *revise* and *modulate her concept image.* Additionally, her graph was discussed by the teacher in the classroom. This student only engaged in the learning cycle when the intrinsic feedback of the tool was combined with extrinsic feedback of a peer student and from the teacher.

Conclusions and discussion

Learning dynamic graphs at secondary school represent a challenge for many students. In this study we investigate how and why the IVM-tool enabled students to improve (or not) their graphical representation of a dynamic event. We have seen that all nine students failed to produce a correct graph in a first trial with the tool. Five of these students could improve their initial graph and produced a correct graph in their subsequent trial. The features of the tool that supported them the most were to see the bottle-form correspondent to their graph at the end (reward-feature) and the opportunity to self-construct the graph, because it challenged them to think and try to imagine how the graph it would be. One student referred also to the animation 3D, which have encouraged him to imagine better “how the water was distributed” in the bottle and with relation to the graph. Three other students have tried improving their initial graph with the tool and although they could not produce a correct graph the tool seemed to have helped them to improve their thinking, as they became aware that the graph should not be linear or contain linear parts. They conclude this because of the reward-feature and in combination with other features (comparison-task or with the animation help-3D). One student only improved her graph when this was combined with extrinsic feedback from peer-student and the teacher.

Reflecting upon these results, we realize that even though the tool showed potential to engage students in thinking about mathematical graphs this was insufficient for all the students to generate an appropriate representation. These students realized with the help of the tool what they have done wrong and the feedback maintained them on the task. But the students did not successfully use these and other features of the tool to move forwards. Why did the tool not help the students? Analysis with the covariational framework suggests the three students possessed a limited understanding of covariational reasoning. What can seem to be a reason for students ’difficulty in imagining the change of the height varying as function of the volume and (Thompson, 2011; Carlson et al., 2010). The tool contains two animations that were explicitly designed for students to engage in covariational thinking: the animation 3D and the interactive animation. But as our results showed none of the students consulted it or they only did it superficially. We cannot therefore comprehend how these features could have supported the students. Another feature of the tool that we had expected to help the students was the reward-feature and, in a certain way it did. The feedback assisted students to realize they were not constructing the correct graph but did not provide other directions or explanations that help them forwards. This is an aspect that requires further thought and investigation. In addition, expanding the tool with more tasks and different contexts is needed in order to offer students enough opportunities for exploring and practice covariational thinking and reasoning.

This study extends previous research about learning dynamic graphs and covariational reasoning. It adds knowledge about students thinking in contexts of change and their difficulties in constructing graphs from themselves. The results provide directions to improve the instructional environment of IVM that can be useful for teachers and researchers interested in using the tool. Further research is needed to gain more insight in the help-features and to improve the intrinsic feedback. We also need to investigate the learning with the tool in larger settings: with more students, teachers and whole classrooms. The prototypical version of the tool presented at CERME 11 is available at https://app.dwo.nl/dwo/apps/player.html#570660.

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